

# Archetype Summary

to accompany

## A First Course in Linear Algebra

by

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|                    | A | B  | C | D | E | F   | G | H | I | J | K  | L | M | N | O | P | Q | R | S | T | U | V  | W  | X |
|--------------------|---|----|---|---|---|-----|---|---|---|---|----|---|---|---|---|---|---|---|---|---|---|----|----|---|
| Type               | S | S  | S | S | S | S   | S | S | S | S | M  | M | L | L | L | L | L | L | L | L | L | L  | L  | L |
| Vars, Cols, Domain | 3 | 3  | 4 | 4 | 4 | 4   | 2 | 2 | 7 | 9 | 5  | 5 | 5 | 5 | 3 | 3 | 5 | 5 | 3 | 5 | 6 | 4  | 3  | 4 |
| Eqns, Rows, CoDom  | 3 | 3  | 3 | 3 | 3 | 4   | 5 | 4 | 4 | 6 | 5  | 5 | 3 | 3 | 5 | 5 | 5 | 5 | 4 | 6 | 4 | 4  | 3  | 4 |
| Solution Set       | I | U  | I | I | N | U   | U | N | I | I |    |   |   |   |   |   |   |   |   |   |   |    |    |   |
| Rank               | 2 | 3  | 3 | 2 | 2 | 4   | 2 | 2 | 3 | 4 | 5  | 3 | 2 | 3 | 2 | 3 | 4 | 5 | 2 | 5 | 4 | 4  | 3  | 3 |
| Nullity            | 1 | 0  | 1 | 2 | 2 | 0   | 0 | 0 | 4 | 5 | 0  | 2 | 3 | 2 | 1 | 0 | 1 | 0 | 1 | 0 | 2 | 0  | 0  | 1 |
| Injective          |   |    |   |   |   |     |   |   |   |   |    |   | X | X | N | Y | N | Y | N | Y | X | Y  | Y  | N |
| Surjective         |   |    |   |   |   |     |   |   |   |   |    |   | N | Y | X | X | N | Y | X | X | Y | Y  | Y  | N |
| Full Rank          | N | Y  | Y | N | N | Y   | Y | Y | N | N | Y  | N |   |   |   |   |   |   |   |   |   |    |    |   |
| Nonsingular        | N | Y  | Y |   |   | Y   | Y | Y |   |   | Y  | N |   |   |   |   |   |   |   |   |   | Y  | Y  | N |
| Invertible         | N | Y  | Y |   |   | Y   | Y | Y |   |   | Y  | N |   |   |   |   | N | Y |   |   |   | -2 | -3 | 0 |
| Determinant        | 0 | -2 |   |   |   | -18 |   |   |   |   | 16 | 0 |   |   |   |   |   |   |   |   |   |    |    | Y |
| Diagonalizable     | N | Y  |   |   |   | Y   |   |   |   |   | Y  | Y |   |   |   |   |   |   |   |   |   |    |    | Y |

### Archetype Facts

S=System of Equations, M=Matrix, L=Linear Transformation  
U=Unique solution, I=Infinitely many solutions, N=No solutions  
Y=Yes, N=No, X=Impossible, blank=Not Applicable

### Archetype A

$$\begin{aligned}x_1 - x_2 + 2x_3 &= 1 \\2x_1 + x_2 + x_3 &= 8 \\x_1 + x_2 &= 5\end{aligned}$$

### Archetype B

$$\begin{aligned}-7x_1 - 6x_2 - 12x_3 &= -33 \\5x_1 + 5x_2 + 7x_3 &= 24 \\x_1 + 4x_3 &= 5\end{aligned}$$

### Archetype C

$$\begin{aligned}2x_1 - 3x_2 + x_3 - 6x_4 &= -7 \\4x_1 + x_2 + 2x_3 + 9x_4 &= -7 \\3x_1 + x_2 + x_3 + 8x_4 &= -8\end{aligned}$$

### Archetype D

$$\begin{aligned}2x_1 + x_2 + 7x_3 - 7x_4 &= 8 \\-3x_1 + 4x_2 - 5x_3 - 6x_4 &= -12 \\x_1 + x_2 + 4x_3 - 5x_4 &= 4\end{aligned}$$

### Archetype E

$$\begin{aligned}2x_1 + x_2 + 7x_3 - 7x_4 &= 2 \\-3x_1 + 4x_2 - 5x_3 - 6x_4 &= 3 \\x_1 + x_2 + 4x_3 - 5x_4 &= 2\end{aligned}$$

### Archetype F

$$\begin{aligned}33x_1 - 16x_2 + 10x_3 - 2x_4 &= -27 \\99x_1 - 47x_2 + 27x_3 - 7x_4 &= -77 \\78x_1 - 36x_2 + 17x_3 - 6x_4 &= -52 \\-9x_1 + 2x_2 + 3x_3 + 4x_4 &= 5\end{aligned}$$

## Archetype G

$$2x_1 + 3x_2 = 6$$

$$-x_1 + 4x_2 = -14$$

$$3x_1 + 10x_2 = -2$$

$$3x_1 - x_2 = 20$$

$$6x_1 + 9x_2 = 18$$

## Archetype H

$$2x_1 + 3x_2 = 5$$

$$-x_1 + 4x_2 = 6$$

$$3x_1 + 10x_2 = 2$$

$$3x_1 - x_2 = -1$$

$$6x_1 + 9x_2 = 3$$

## Archetype I

$$x_1 + 4x_2 - x_4 + 7x_6 - 9x_7 = 3$$

$$2x_1 + 8x_2 - x_3 + 3x_4 + 9x_5 - 13x_6 + 7x_7 = 9$$

$$2x_3 - 3x_4 - 4x_5 + 12x_6 - 8x_7 = 1$$

$$-x_1 - 4x_2 + 2x_3 + 4x_4 + 8x_5 - 31x_6 + 37x_7 = 4$$

## Archetype J

$$x_1 + 2x_2 - 2x_3 + 9x_4 + 3x_5 - 5x_6 - 2x_7 + x_8 + 27x_9 = -5$$

$$2x_1 + 4x_2 + 3x_3 + 4x_4 - x_5 + 4x_6 + 10x_7 + 2x_8 - 23x_9 = 18$$

$$x_1 + 2x_2 + x_3 + 3x_4 + x_5 + x_6 + 5x_7 + 2x_8 - 7x_9 = 6$$

$$2x_1 + 4x_2 + 3x_3 + 4x_4 - 7x_5 + 2x_6 + 4x_7 - 11x_9 = 20$$

$$x_1 + 2x_2 + 5x_4 + 2x_5 - 4x_6 + 3x_7 + 8x_8 + 13x_9 = -4$$

$$-3x_1 - 6x_2 - x_3 - 13x_4 + 2x_5 - 5x_6 - 4x_7 + 13x_8 + 10x_9 = -29$$

### Archetype K

$$\begin{bmatrix} 10 & 18 & 24 & 24 & -12 \\ 12 & -2 & -6 & 0 & -18 \\ -30 & -21 & -23 & -30 & 39 \\ 27 & 30 & 36 & 37 & -30 \\ 18 & 24 & 30 & 30 & -20 \end{bmatrix}$$

### Archetype L

$$\begin{bmatrix} -2 & -1 & -2 & -4 & 4 \\ -6 & -5 & -4 & -4 & 6 \\ 10 & 7 & 7 & 10 & -13 \\ -7 & -5 & -6 & -9 & 10 \\ -4 & -3 & -4 & -6 & 6 \end{bmatrix}$$

### Archetype M

$$T: \mathbb{C}^5 \mapsto \mathbb{C}^3, \quad T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \right) = \begin{bmatrix} x_1 + 2x_2 + 3x_3 + 4x_4 + 4x_5 \\ 3x_1 + x_2 + 4x_3 - 3x_4 + 7x_5 \\ x_1 - x_2 - 5x_4 + x_5 \end{bmatrix}$$

### Archetype N

$$T: \mathbb{C}^5 \mapsto \mathbb{C}^3, \quad T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \right) = \begin{bmatrix} 2x_1 + x_2 + 3x_3 - 4x_4 + 5x_5 \\ x_1 - 2x_2 + 3x_3 - 9x_4 + 3x_5 \\ 3x_1 + 4x_3 - 6x_4 + 5x_5 \end{bmatrix}$$

### Archetype O

$$T: \mathbb{C}^3 \mapsto \mathbb{C}^5, \quad T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} -x_1 + x_2 - 3x_3 \\ -x_1 + 2x_2 - 4x_3 \\ x_1 + x_2 + x_3 \\ 2x_1 + 3x_2 + x_3 \\ x_1 + 2x_3 \end{bmatrix}$$

### Archetype P

$$T: \mathbb{C}^3 \mapsto \mathbb{C}^5, \quad T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \right) = \begin{bmatrix} -x_1 + x_2 + x_3 \\ -x_1 + 2x_2 + 2x_3 \\ x_1 + x_2 + 3x_3 \\ 2x_1 + 3x_2 + x_3 \\ -2x_1 + x_2 + 3x_3 \end{bmatrix}$$

### Archetype Q

$$T: \mathbb{C}^5 \mapsto \mathbb{C}^5, \quad T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \right) = \begin{bmatrix} -2x_1 + 3x_2 + 3x_3 - 6x_4 + 3x_5 \\ -16x_1 + 9x_2 + 12x_3 - 28x_4 + 28x_5 \\ -19x_1 + 7x_2 + 14x_3 - 32x_4 + 37x_5 \\ -21x_1 + 9x_2 + 15x_3 - 35x_4 + 39x_5 \\ -9x_1 + 5x_2 + 7x_3 - 16x_4 + 16x_5 \end{bmatrix}$$

### Archetype R

$$T: \mathbb{C}^5 \mapsto \mathbb{C}^5, \quad T \left( \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \right) = \begin{bmatrix} -65x_1 + 128x_2 + 10x_3 - 262x_4 + 40x_5 \\ 36x_1 - 73x_2 - x_3 + 151x_4 - 16x_5 \\ -44x_1 + 88x_2 + 5x_3 - 180x_4 + 24x_5 \\ 34x_1 - 68x_2 - 3x_3 + 140x_4 - 18x_5 \\ 12x_1 - 24x_2 - x_3 + 49x_4 - 5x_5 \end{bmatrix}$$

### Archetype S

$$T: \mathbb{C}^3 \mapsto M_{22}, \quad T \left( \begin{bmatrix} a \\ b \\ c \end{bmatrix} \right) = \begin{bmatrix} a - b & 2a + 2b + c \\ 3a + b + c & -2a - 6b - 2c \end{bmatrix}$$

### Archetype T

$$T: P_4 \mapsto P_5, \quad T(p(x)) = (x - 2)p(x)$$

### Archetype U

$$T: M_{23} \mapsto \mathbb{C}^4, \quad T\left(\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\right) = \begin{bmatrix} a + 2b + 12c - 3d + e + 6f \\ 2a - b - c + d - 11f \\ a + b + 7c + 2d + e - 3f \\ a + 2b + 12c + 5e - 5f \end{bmatrix}$$

### Archetype V

$$T: P_3 \mapsto M_{22}, \quad T(a + bx + cx^2 + dx^3) = \begin{bmatrix} a + b & a - 2c \\ d & b - d \end{bmatrix}$$

### Archetype W

$$T: P_2 \mapsto P_2, \quad T(a + bx + cx^2) = (19a + 6b - 4c) + (-24a - 7b + 4c)x + (36a + 12b - 9c)x^2$$

### Archetype X

$$T: M_{22} \mapsto M_{22}, \quad T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \begin{bmatrix} -2a + 15b + 3c + 27d & 10b + 6c + 18d \\ a - 5b - 9d & -a - 4b - 5c - 8d \end{bmatrix}$$